Paper Reference(s)

6666

Edexcel GCE

Pure Mathematics C4

Advanced Level

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination
Answer Book (AB16)
Mathematical Formulae (Lilac)
Graph Paper (ASG2)

Items included with question papers
Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI-89, TI-92, Casio CFX-9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics C4), the paper reference (6666), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. Use the binomial theorem to expand $(4 - 3x)^{-\frac{1}{2}}$, in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

    (5)

---

2. The curve C has equation

    $$13x^2 + 13y^2 - 10xy = 52.$$

    Find an expression for $\dfrac{dy}{dx}$ as a function of x and y, simplifying your answer.

    (6)

---

3. Use the substitution $x = \tan \theta$ to show that

    $$\int_0^1 \frac{1}{(1 + x^2)^2}\, dx = \frac{\pi}{8} + \frac{1}{4}\ .$$

    (8)

4.

Figure 1

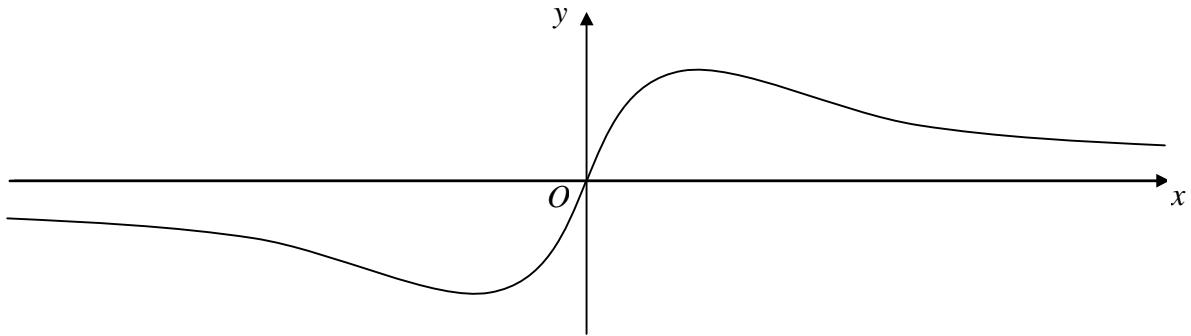

Figure 1 shows part of the curve with parametric equations

$$x = \tan t, \quad y = \sin 2t, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

(a) Find the gradient of the curve at the point P where $t = \frac{\pi}{3}$.

(4)

(b) Find an equation of the normal to the curve at P.

(3)

(c) Find an equation of the normal to the curve at the point Q where $t = \frac{\pi}{4}$.

(2)

5. The vector equations of two straight lines are

$\mathbf{r} = 5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and

$\mathbf{r} = 2\mathbf{i} - 11\mathbf{j} + a\mathbf{k} + \mu(-3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$.

Given that the two lines intersect, find

(a) the coordinates of the point of intersection,

(5)

(b) the value of the constant a,

(2)

(c) the acute angle between the two lines.

(4)

---

6. Given that
$$\frac{11x-1}{(1-x)^2(2+3x)} \equiv \frac{A}{(1-x)^2} + \frac{B}{(1-x)} + \frac{C}{(2+3x)},$$

(a) find the values of A, B and C.

(4)

(b) Find the exact value of $\displaystyle\int_0^{\frac{1}{2}} \frac{11x-1}{(1-x)^2(2+3x)}\,dx$, giving your answer in the form $k + \ln a$, where

k is an integer and a is a simplified fraction.

(7)

Turn over

7. (a) Given that $u = \dfrac{x}{2} - \dfrac{1}{8} \sin 4x$, show that $\dfrac{du}{dx} = \sin^2 2x$.

(4)
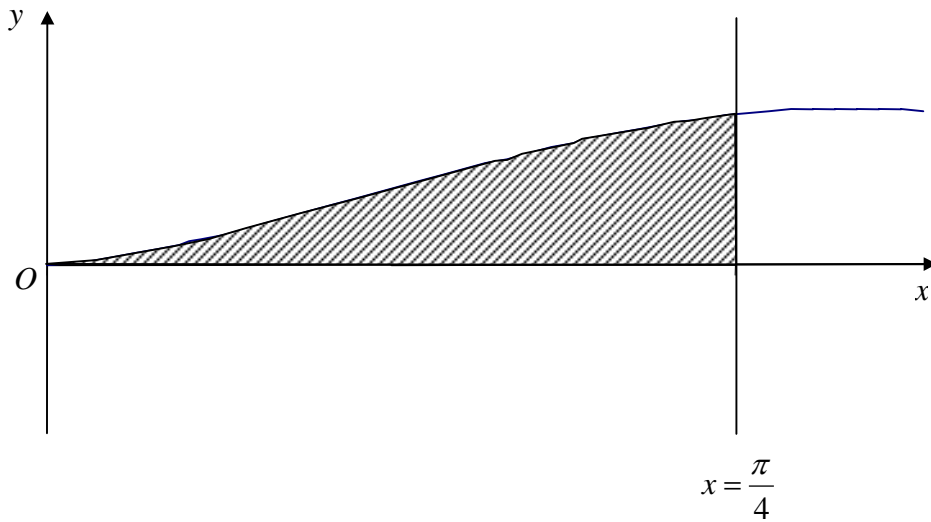
Figure 2

$$x = \frac{\pi}{4}$$

Figure 2 shows the finite region bounded by the curve $y = x^{\frac{1}{2}} \sin 2x$, the line $x = \dfrac{\pi}{4}$ and the x-axis. This region is rotated through 2π radians about the x-axis.

(b) Using the result in part (a), or otherwise, find the exact value of the volume generated.

(8)

8. A circular stain grows in such a way that the rate of increase of its radius is inversely proportional to the square of the radius. Given that the area of the stain at time t seconds is A cm^2,

(a) show that $\dfrac{\mathrm{d}A}{\mathrm{d}t} \propto \dfrac{1}{\sqrt{A}}$.

(6)

Another stain, which is growing more quickly, has area S cm^2 at time t seconds. It is given that

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{2\mathrm{e}^{2t}}{\sqrt{S}}.$$

Given that, for this second stain, $S = 9$ at time $t = 0$,

(b) solve the differential equation to find the time at which $S = 16$. Give your answer to 2 significant figures.

(7)

END

Turn over